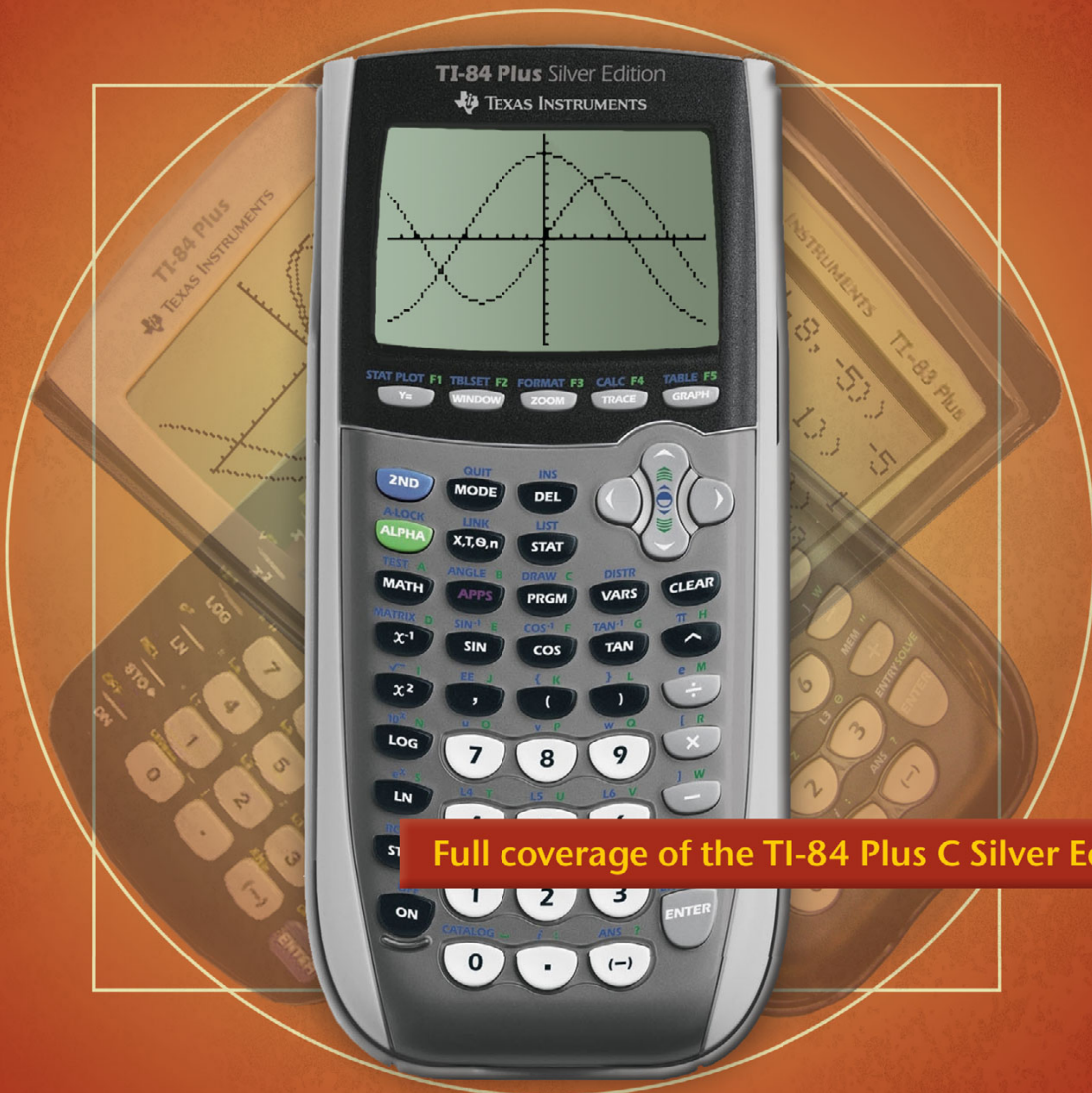
Sample Chapter

Using the TI-83 Plus/TI-84 Plus



Full coverage of the TI-84 Plus C Silver Edition

Christopher R. Mitchell

MANNING

Using the TI-83 Plus/TI-84 Plus

by Christopher R. Mitchell

Chapter 5

brief contents

5 Expanding your graphing skills

This chapter covers

- Working with Parametric, Polar, and Sequence graphing modes
- Drawing diagrams and annotating graphs
- Saving and restoring graph snapshots and graph settings

Graphing is one of the key features of your graphing calculator; chapter 3 pointed out that it's so important that it's half of the device's name. In that chapter, we worked with normal Cartesian graphing, where you visualize equations like $y = x^2+1$. In this chapter, you'll see many of the advanced graphing features your calculator includes, such as new graphing modes and annotating graphs with drawings.

Why would you want to graph in other schemes than Cartesian graphing, anyway? The short answer is that there are many functions and graphs that you can't represent in the form $y = f(x)$, which defines the vertical position (y) of each point in the graph by passing its horizontal position (x) through a function. Figure 5.1 shows the four different types of graph coordinate systems that your calculator can work with.

Whereas chapter 3 taught you rectangular or Cartesian graphing, this chapter will introduce the other three modes in figure 5.1. We'll first go through Parametric

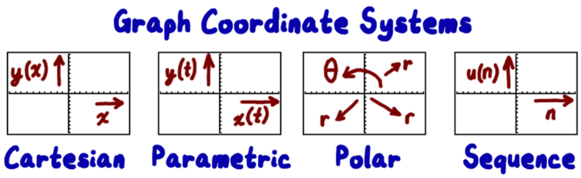


Figure 5.1 The four types of graph coordinate systems your calculator knows. At far left, Cartesian (or rectangular) coordinates, where *x* is the independent variable and *y*(*x*) is the dependent variable. Parametric mode at center-left makes both *x* and *y* dependent on a third independent variable, *t*, which lets you graph functions that can't be expressed as Cartesian functions in the form *y*(*x*). At center-right, polar coordinates, where the radius *r* is dependent on the independent variable θ (angle). Finally, sequence graphing is for normal or recursive functions applied to independent *n* values.

(PAR) mode, including some of the types of graphs it makes possible. Next, I'll show you Polar (POL) mode, touching on the graph variables used for polar graphing and examples of polar functions. The third and most esoteric type of graph you'll learn is Sequence (SEQ) mode. With all four of the graphing modes your calculator supports under your belt, we'll move on to drawing.

You can annotate any kind of graph with lines, points, text, and sketches, as you'll learn in the latter half of this chapter. You can save annotated graphs to later recall. Many enterprising (and bored) students have even used the drawing tools to doodle or draw diagrams on their calculators. If you have a bunch of graphed equations that you need to use later, you can save and recall those too, as I'll show you.

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GRAPH STYLES ON THE TI-84 PLUS C SILVER EDITION In chapter 3, you learned to set the color and line style of rectangular (Function mode) graphs. The same skills apply to parametric, polar, and sequence graphs. You can change graphs' lines to any of 7 different styles and 15 different colors.

Let's get started with Parametric mode, which is the closest of the three new graphing modes to the rectangular graphing you're now familiar with.

5.1 Parametric mode

You've learned all about graphing functions that map *x* values to *y* values, such as $Y_1=2+X$ or $Y_2=\sin(3X/5)$. One of the most noticeable shortcomings is that you can have only one *y* value for every *x* value. Imagine a parabola, like the one in the left screenshot in figure 5.2. If you were to draw a vertical line through any *x* value, you'd hit only one point on the graph, whereas a horizontal line might hit zero, one, or two points on the graph. Rectangular mode can only create graphs like those in the left screenshot, where any vertical line crosses a graphed function at one or zero points.

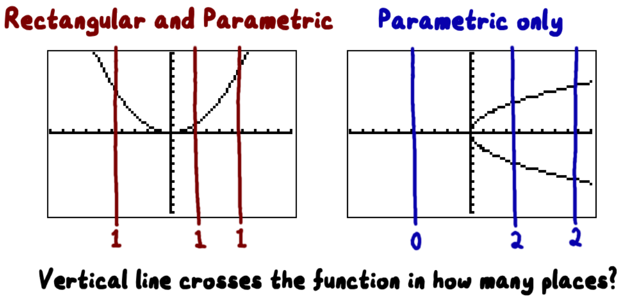

Figure 5.2 The power of parametric graphing. Rectangular graphing can map only one *y* value to each *x* value, meaning that any vertical line will cross a graphed function in exactly one place (or zero, if there is a discontinuity). Parametric mode can graph anything Rectangular mode can, but it can also draw graphs with multiple (or zero) *y* values for each *x*. The sideways parabola on the right side is one such example.

The right side of figure 5.2 shows an example of what Parametric mode can do with ease, although Rectangular mode can't manage it. The graph shown is essentially the equation `X=(Y/2)²`, which is the `Y=(X/2)²` graph shown on the left turned on its side. Parametric mode is only one of two ways to graph equations of the form $x = f(y)$, in addition to being able to graph circles, functions that trace over themselves many times, and much more.

Parametric graphing in a nutshell

- To switch to Parametric mode, press MODE, move the cursor to `PAR`, and press ENTER.
- When you go to the Y= menu, you'll see pairs of functions you can enter, such as X_{1T} and Y_{1T}. You must enter equations in pairs.
- When you press the XTθn key in `PAR`, it will type a `T` instead of an `X`, because `T` is the independent variable for parametric graphing.
- As in normal (Function/Rectangular) mode, in Polar mode the Zoom menu options discussed in section 3.3.1 can be used to adjust what you see in the graph.
- To adjust the range and granularity of `T` values plugged into the parametric functions, use the Window menu.

You can see that parametric graphing is powerful, but how exactly can it do what it does? What gives it its power? Your favorite math book or teacher can give you more details, but the essentials are that every point in a parametric graph is defined by two functions, not just one. In rectangular graphing, as shown on the left in figure 5.3, you take each *x* value and plug it into a single function (written as $y = f(x)$), and the *y* value pops out. By definition, each *x* value can have only a single *y* value, because a function can't give two different *y* values when the same *x* value is plugged in.

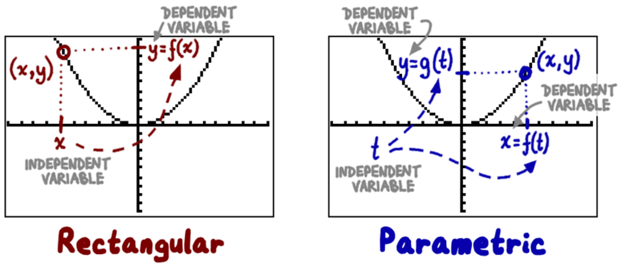

Figure 5.3 The mechanical differences between rectangular and parametric graphing. With rectangular graphing, every *y* value is generated by passing a corresponding *x* value through the function to be graphed. Parametric graphing is much more powerful because graphed functions can pass through the same *x* coordinates, *y* coordinates, and even (*x*,*y*) point multiple times.

Parametric graphing, by contrast, defines every point on the graph by *two* functions; call them $x = f(t)$ and $y = g(t)$. By making both x and y depend on t but not on each other, you can express many more graphs. You can have graphs with multiple x values for the same y, multiple y values for the same x, and even graphs that intersect with themselves or trace over themselves.

If you have time for a longer introduction than the "Parametric graphing in a nutshell" sidebar, let me lead you through two parametric graphing examples. The first will show you how to graph a circle and then modify the `T` values plugged in to draw a semicircle. The second exercise will demonstrate graphing a Lissajous curve ("*LEASE-a-ju*"), a fancy family of graphs that can only be drawn in Parametric mode.

5.1.1 Parametric example: graphing a circle

Graphing a circle is a challenge in Function mode but surprisingly easy in Parametric mode. It's so easy because if you have an angle (call it t, for example) and a radius (call it r), then the (x,y) position of the point r units from (0,0) at angle t is simply

$$x = r\cos(t)$$
$$y = r\sin(t)$$

Because parametric graphs work perfectly with equations for x and y that are both in terms of a third independent variable t, you could use this to draw a circle. If you pick a constant radius, say $r = 6$, and plug in every possible t (angle) value, you'll get points forming a circle around the origin (0,0).

You can apply this almost exactly as written to graphing a circle in Parametric mode on your graphing calculator. Enter `X1T=6cos(T)` and `Y1T=6sin(T)`, after making sure you set your calculator to Parametric mode (see the "Parametric graphing in a nutshell" sidebar). The equations should look like the left screenshot in figure 5.4. Remember, [XTθn] types `T` when you're in Parametric mode. Press [GRAPH], and your calculator should draw a circle, as illustrated on the right in figure 5.4.

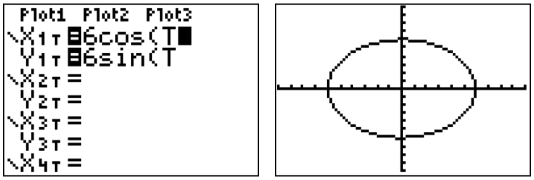

Figure 5.4 Graphing a circle of radius 6 in Parametric mode. On the left, entering the `X` and `Y` equations that define the circle, as described in the text; on the right, the results of graphing these equations.

My circle isn't round. Figure 5.4 shows a circle that looks more like an ellipse. As mentioned in chapter 3, your calculator's default zoom sets `Xmin` and `Ymin` to –10 and `Xmax` and `Ymax` to 10. But because the screen is wider than it is tall, this makes circles look stretched horizontally. You can fix this (for any Zoom setting!) by pressing ZOOM and choosing `5:ZSquare`.

If anything looks wrong, and the graph doesn't look like the right side of figure 5.4, press ZOOM. Choose the trusty `6:ZStandard` option that even in Parametric mode resets graph settings to sane defaults. In Parametric mode, `ZStandard` sets the same graph edges as in Rectangular mode, namely `Xmin` = –10, `Xmax` = 10, `Ymin` = –10, and `Ymax` = 10. It does something else—it sets `Tmin`, `Tmax`, and `Tstep` (found under WINDOW):

- To graph a pair of functions like X_{1T} and Y_{1T}, your calculator plugs `T` values between `Tmin` and `Tmax` into the pair of functions. By default, `Tmin` = 0 and `Tmax` = 2π.
- Your calculator can't plug in *every* value of `T` between `Tmin` and `Tmax`, because there are infinitely many. The `Tstep` value tells the calculator how much to add to `T` each time it plugs in a new value. By default, `Tstep` is π/24.

You can adjust the `Tmin`, `Tstep`, and `Tmax` values to change how a parametric graph looks. I'll show you how changing `Tmax` can turn the circle you graphed into a semicircle.

Graphing a partial circle

You may recall from trigonometry or geometry that a circle is 2π radians (or 360°) around. Because by default `Tmin` = 0 and `Tmax` = 2π, your calculator plugs all the values necessary into your X_{1T}`=6cos(T)` and Y_{1T}`=6sin(T)` equations to draw a full circle of radius 6. If a full circle is 2π, it stands to reason that half of a circle is just π. Press WINDOW, move the cursor down to `Tmax=`, and change the value to π (which you can type with 2nd ^). The left screenshot in figure 5.5 shows what you'll see. Press ENTER to set the value, and the screen will transform into what you see in the center in figure 5.5.

Press GRAPH to graph the result: a semicircle that should match the right side in figure 5.5. Your calculator plugs in `T` values from `T` = 0 to `T` = π, and you get the first half of the circle. You can change `Tmin` to π and `Tmax` to 2π if you want the other half of the circle. You can also try making `Tstep` smaller and larger. Notice that if you make it larger, the circle gets rougher, because your calculator is plugging in fewer `T` values. Make it smaller, and the circle gets smoother but also takes longer to graph. If you set it low enough, you might even be able to draw polygons!

We can revisit parabolic or projectile motion from chapter 3 with parametric graphing, this time throwing a ball in two dimensions.

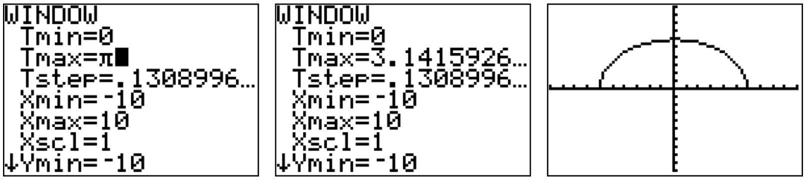

Figure 5.5 Modify the Window settings to graph a semicircle instead of a circle. Press WINDOW to access the menu shown at left and center, where you can modify `Tmin`, `Tmax`, or `Tstep`. Once you store your changes and press GRAPH, you'll see the modified graph, such as the semicircle at right.

5.1.2 Parametric example: throwing a baseball

In section 3.2.1, we used Rectangular graphing mode to look at what would happen when you threw a ball into the air. You graphed time on the x axis and the height of the ball on the y axis, and observed the height of the ball from the time you threw it straight upward until it fell back to the ground. If you didn't throw the ball straight up but instead lobbed it across a field, the problem would get more complicated. Luckily, because Parametric mode lets you graph x and y as a function of t, you can use it to easily graph the path of a thrown baseball. This example will show you how.

Chapter 3 taught you that the equation for the height of a thrown ball looks like this:

$$y = y_0 + v_0x + 0.5ax^2$$

In this equation, a is the acceleration due to gravity (–9.8 meters/second^2), y_0 is the initial height of the ball when thrown, v_0 is how fast it was going (in meters/second) when you threw it, x is time, and y is the height of the ball at time x (in seconds). If you switch to the more intuitive variable t for time, you can use x for the horizontal position at time t and make x_0 represent the starting horizontal position of the ball. If you threw a ball across a field with an initial velocity v_0, here's what the two equations describing its motion would look like:

$$x = x_0 + v_{0x}t$$
$$y = y_0 + v_{0y}t + 0.5at^2$$

GRAPHING A BASEBALL'S PATH

You can use these equations to graph the path of a thrown baseball over time. A professional baseball player might be able to throw a ball at 90 miles per hour, or about 40m/s. Let's say he throws it at a 10° angle to the ground, as shown in figure 5.6. Physics and trigonometry tell us that the x-component and y-component of that initial velocity, shown in figure 5.6, are

$$v_{0x} = v_0 \cos(10°) = 40*0.985 = 39.39 \text{ m/s}$$
$$v_{0y} = v_0 \sin(10°) = 40*0.174 = 6.95 \text{ m/s}$$

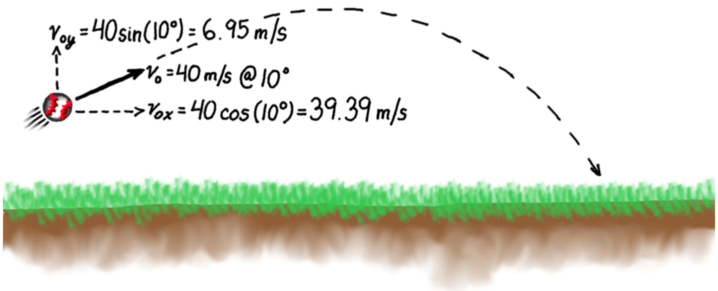

Figure 5.6 Throwing a baseball at a 10-degree angle to the ground and seeing how to calculate the *x*- and *y*-components of the initial velocity of 40 m/s

The math to calculate these components is shown at left in figure 5.7, along with the equations you'll soon enter.

You can create a pair of parametric equations describing the motion of the baseball in figure 5.6 by plugging v_{0x} and v_{0y} for this ball into the equations for x and y and substituting x_0, y_0, and a as well. You may recall that acceleration due to gravity, called either g or a, is –9.8 m/s^2 (negative because it's acceleration downward). To make things easy, let's assume that the ball starts at horizontal position x_0 = 0 meters, and that if the person throwing the baseball is about 2 meters tall, the ball's height when he releases it is y_0 = 1.8m:

$$x = x_0 + v_{0x}t = 0 + 39.39t = 39.39t$$
$$y = y_0 + v_{0y}t + 0.5at^2 = 1.8 + 6.95t - 4.9t^2$$

Enter these equations as `X1T` and `Y1T`. Press [Y=], and if there are any equations already entered, clear them or disable them. If you don't see pairs of parametric equation prompts, as at center in figure 5.7, go to the Mode menu and make sure you're in Polar mode.

> **DEGREE SYMBOL?** Figure 5.7 shows the degree symbol. If you want to use `sin(` and `cos(` without specifying an angle mode in the Mode menu, you can put the ° (degree) or ʳ (radian) symbol after a number to specify whether it's a degree or radian angle. Both symbols are in the Angle menu ([2nd] [APPS]).

You also need to choose a window. Because the ball starts at x_0 = 0, `Xmin`=0 seems like a good choice; and because the ball can't go below the ground, `Ymin`=0 is also logical. You can find the maximum y (height) the ball will reach graphically or by taking the

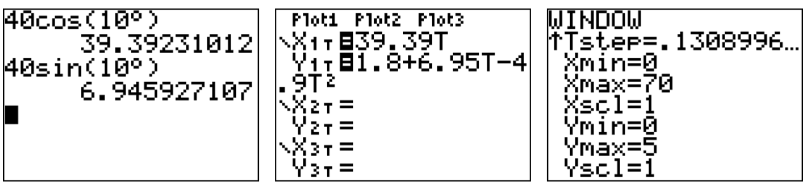

Figure 5.7 Calculating the *x*- and *y*-components of the baseball's initial velocity (left). Entering the equations for the motion of the ball for a parametric plot (center), and a good window to see the function (right).

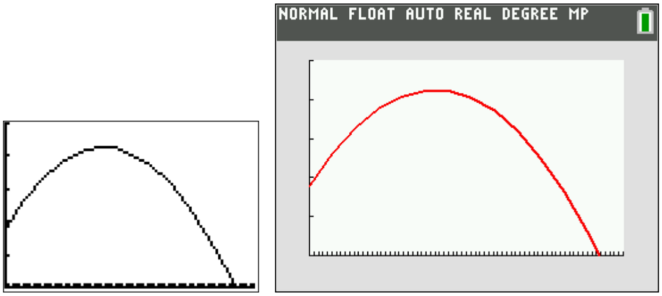

Figure 5.8 Graphing the motion of a ball in two dimensions versus time, on a TI-83 Plus or TI-84 Plus (left), or on a TI-84 Plus C Silver Edition (right)

derivative of the expression for *y*. Try adjusting `Ymax` until you get the graph to fit well, or take the derivative and find the roots. As a hint, `Ymax`=5 works well. You can also plug in 0 for *y* to figure out when the ball will hit the ground, which will tell you a good `Xmax`. Alternatively, you can guess and check. You'll probably end up with `Xmax`=70 or so, as shown at the right in figure 5.7.

Your resulting graph should look like one of the screenshots in figure 5.8. Because the ball started moving with rightward and upward velocity, it curves upward before beginning to fall again. Remember that you're now graphing as if you were standing in the bleachers, watching the ball's movement. You can figure out where the ball was at each instant by tracing over the graph. Press TRACE, and observe the `T`, `X`, and `Y` values shown. Unfortunately, the calculator can't find maxima, minima, or zeroes in Parametric mode, so you'll have to use the Table or calculus to figure out when the baseball reaches the peak of its curve or when it hits the ground.

Let's look at a third and final parametric graphing example: a fancy family of curves called Lissajous curves.

5.1.3 Parametric example: a Lissajous curve

A Lissajous curve, also called a Bowditch curve, is a family of parametric functions created with sine and cosine. Figure 5.9 shows an example of a Lissajous curve drawn on a graphing calculator. You can test it yourself by plugging in the two equations shown on the left in figure 5.9, resulting in the graph shown on the right. As always, press ZOOM and choose `6:ZStandard` if you don't get the same graph.

To give you a bit of background, a Lissajous curve is any of a family of parametric curves of the form

$$x(t) = a\sin(ct + d)$$
$$y(t) = b\sin(et)$$

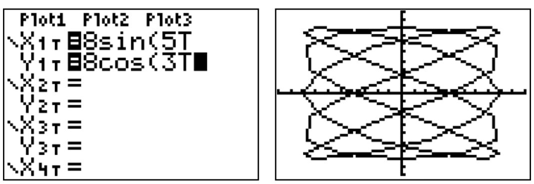

Figure 5.9 Graphing a Lissajous curve in Parametric mode. The left screenshot shows the equations to enter, and the right shows the result.

In these equations, *a*, *b*, *c*, *d*, and *e* are constant numbers, not variables. In the example I just showed you, we picked $a = b = 8$, $c = 5$, $d = 0$, and $e = 3$. You should try fiddling with these variables to get different curves in the family, some of which are unique.

The keen eye might also notice that circles are technically in the Lissajous curve family! If you let $c = e = 1$, $d = \pi/2$, and $a = b$, you'll get a circle of radius *a*. You can also graph an ellipse by keeping $c = e = 1$ and $d = \pi/2$ but making *a* and *b* unequal. The semi-minor radius (the distance from the middle of the ellipse to the top or bottom) will be equal to *a*, and the semi-major radius (the distance from the middle of the ellipse to the left or right side) will be equal to *b*.

IMPROVING PARAMETRIC RESOLUTION

Want to make that curve look smoother? You can apply a lesson I taught you in the parametric circle example: make `Tstep` smaller. Try changing `Tstep` to π/48 by pressing [WINDOW], changing `Tstep`, and then pressing [GRAPH]. The curve should appear much smoother, as in figure 5.10, although it will take longer to render.

Parametric mode is a powerful change of pace, great for expressing functions that are impossible in Rectangular mode. We'll now move on to Polar mode, which offers some secrets of its own.

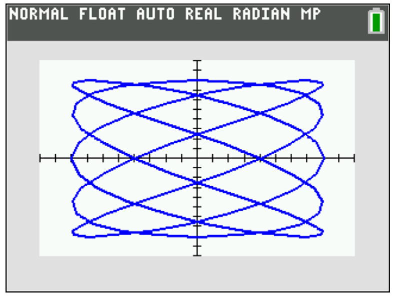

Figure 5.10 A Lissajous curve graphed on a TI-84 Plus C Silver Edition with `Tstep=π/48`

5.2 Graphing polar functions

Polar mode shares some attributes of both function and parametric graphing. As in Function mode, single equations map an independent variable to a dependent variable. Like Parametric mode, Polar mode gives you a way to express graphs that would be impossible in Rectangular mode and that overlap themselves.

In Polar mode, the independent variable is θ (the Greek letter theta, which represents an angle). By default, θ ranges from 0 to 2π. The dependent variable is `r` or radius, a distance from the origin at angle θ. There are six polar equations, numbered r_1 through r_6. Once you switch your calculator into Polar mode, as explained in the "Polar graphing in a nutshell" sidebar, the Y= and Window menus will change to reflect the new variables and equations.

To get you accustomed to polar graphing, let's start with an example of the easiest graph you can draw in Polar mode: a circle. Ironically, it's one of the hardest things to do in Rectangular mode, requiring two equations and careful tweaking of the ΔX variable. Polar mode plugs in θ values from 0 to 2π to any equation you enter, gets a radius, and plots a point a distance *r* from the origin in the θ direction. If you enter a constant number like 4.2 or 8 for a polar equation, then your calculator will draw a circle of that radius. Let's try r_1=7.

The first thing you need to do is make sure your calculator is in Polar mode. Go to the Mode menu, move the cursor to `POL`, and press [ENTER]. You should see what the left

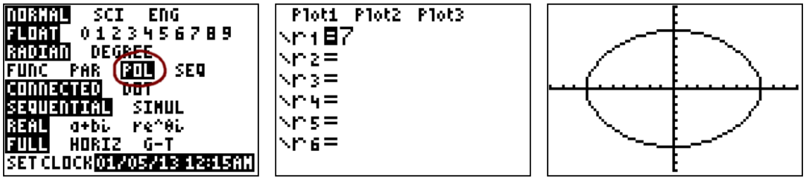

Figure 5.11 Setting Polar mode and graphing your first polar equation. The left shows switching to `POL` in the Mode menu. The center screenshot displays a simple equation to graph, and the right is the resulting circle.

Polar graphing in a nutshell

- To switch to Polar mode, press MODE, move the cursor to `POL`, and press ENTER.
- When you go to the Y= menu, you see six single functions you can enter, from r_1 to r_6.
- When you press the XTθn key in `POL`, it types a θ instead of an `X`, because θ is the independent variable for polar graphing.
- As with Parametric and Function (Rectangular) modes, the Zoom menu options discussed in section 3.3.1 can be used to adjust what you see in the graph.
- To adjust the range and granularity of θ values plugged in to the parametric functions, use the Window menu.

screenshot in figure 5.11 shows. Now press Y=, enter r_1=7 to match the center screenshot in figure 5.11, and press GRAPH to see the result. You'll see what the right screenshot in figure 5.11 shows: a circle of radius 7 centered on the origin. Remember, as we've discussed before, it looks like an ellipse even though it's a circle because your screen isn't square.

If you wanted to fix the confusing shape of the circle, you could press ZOOM and choose the `5:ZSquare` option. In fact, for the other two examples I'm going to show you in this section, I recommend that you use the `ZSquare` setting. Because polar graphing inherently deals with circular things, it's particularly helpful to be able to see polar graphs in proper proportions.

Converting between polar and rectangular coordinates

Although this section teaches you to graph polar equations, you might also want to convert between polar (r, θ) and rectangular (x, y) coordinates. If you want to change whether the graphscreen shows polar or rectangular coordinates when you use the Trace feature, you can switch between `RectGC` and `PolarGC` in the Graph Format (2nd ZOOM) menu.

If you have numerical (r, θ) or (x, y) coordinates, you can convert them to the opposite form on the homescreen using four functions from the Angle menu. All will heed your current Radian/Degree setting in the Mode menu. Press 2nd APPS and use one of these:

(continued)

- R▶Pr(x, y)—Computes r for the polar coordinate form (r, θ) of the rectangular coordinates (x, y)
- R▶Pθ(x, y)—Computes θ for the polar coordinate form (r, θ) of the rectangular coordinates (x, y)
- P▶Rx(r, θ)—Computes x for the rectangular coordinate form (x, y) of the polar coordinates (r, θ)
- P▶Ry(r, θ)—Computes y for the rectangular coordinate form (x, y) of the polar coordinates (r, θ)

To help you get a feel for your calculator's polar graphing features, I want to take you through two examples. The first will show you how to graph a spiral, and the second will introduce a graph called a polar rose. Let's begin with a spiral, a surprisingly easy shape to graph in Polar mode.

5.2.1 Polar example: a spiral

Graphing a spiral might seem rather impossible at first, but Polar mode makes it simple. If you start with a circle, which has a constant radius, then to make a spiral, you can break the circle and pull one end into the center. A first attempt at a spiral might be `r1=θ`, so that when $\theta = 0$, $r = 0$; when $\theta = \pi$, $r = \pi$; and when $\theta = 2\pi$, $r = 2\pi$. In other words, as θ increases, the radius `r` increases as well.

The initial `r1=θ` spiral gets very far from the origin very quickly. You can make it a tighter spiral by dividing θ by a constant, say π, giving you `r1=θ/π`. The left screenshot in figure 5.12 shows entering this equation, and the right screenshot illustrates what happens when you press the GRAPH key (note that this screenshot was taken with `ZStandard` rather than `ZSquare`).

That's not much of a spiral! How can you make it continue? Remember that by default θ goes from 0 to π, so you get only one revolution of the spiral around the origin. To create more of the spiral, you need to make `θmax` larger. First, press WINDOW and change `θmax` to 6π. This will give you three revolutions of the spiral, which should look much more interesting. To make the spiral look even better, switch to a square window: press ZOOM and select the `5:ZSquare` option, if you didn't already have a square window. Now tap GRAPH once more, and you should see the attractive graph in figure 5.13.

You can reverse the direction of the spiral by setting `r1=⁻θ/π` instead. You can make the spiral tighter or looser by adjusting the scaling factor (here $1/\pi$). You can modify `θmin`, `θstep`, and `θmax` to see more or less spiral or to make your calculator graph a smoother spiral. I encourage you to experiment and see what you can do with the graph.

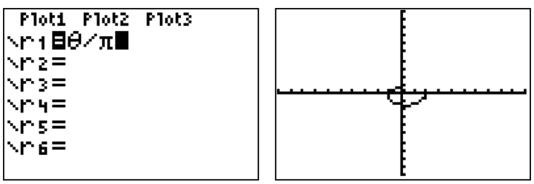

Figure 5.12 An initial attempt at drawing a spiral. With the default graph settings, you get only one section of the spiral from θ = 0 to θ = 2π.

Let's examine another polar exercise: a family of graphs that creates shapes called polar roses.

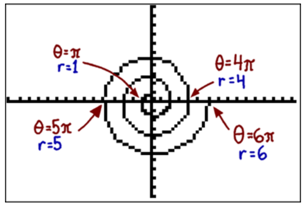

Figure 5.13 Your final spiral, from $\theta = 0$ to $\theta = 6\pi$. Notice the annotated points on the graph, showing the (θ, *r*) coordinates at several points along the spiral.

5.2.2 *Polar example: polar rose*

The *polar rose* is a family of graphs that, as you might guess from the name, are drawn in Polar mode and look like roses. Specifically, they're centered on the origin and have a number of petals. For the sake of this discussion, the general equation for a polar rose is $r(\theta) = a \sin(b\theta)$. The a term controls the size of the rose and is roughly equivalent to a radius from the middle of the rose to the tip of any petal. The b term controls how many petals the rose has:

- If b is even, any rose graphed from $r(\theta) = a\sin(b\theta)$ has $2b$ petals: if $b = 2$, it has 4 petals, and if $b = 5$, it has 10 petals.
- If b is odd, the rose has b petals: if $b = 3$, it has 3 petals, and if $b = 7$, it has 7 petals.

I'll explain how you can graph polar roses, but it's up to you to decide how many petals you want. A few of the possible roses with even numbers of petals are shown in figure 5.14.

First, pick your number of petals. Use the preceding rules about odd and even values of b to decide what value b needs to have. I recommend a radius of $a = 8$, but you're welcome to choose any radius you want. For the following discussion, I'll assume values of $a = 8$ and $b = 4$. Here's how you graph the $r(\theta) = 8\sin(4\theta)$ rose:

1. Make sure you're in Polar mode. If necessary, choose `ZStandard` and then `ZSquare` from the Zoom menu to set the edges of the graph to good values.
2. Press [Y=], clear out any existing equations, and enter `r1=8sin(4θ)`. The easiest way to type the θ variable is to press the [XTθn] key, as long as you're in Polar mode.
3. Press [GRAPH], and your rose will appear.

As in previous exercises, I encourage you to play around with different graph settings and values for the rose equation, if you have time. Tweak a and b to see how the rose changes, and try varying `θstep` to make the rose petals smoother or rougher.

Now you've worked with two new graphing modes in this chapter, and you know how to use three of the four modes your calculator offers. The fourth and final mode, Sequence, is the least used but still powerful and handy.

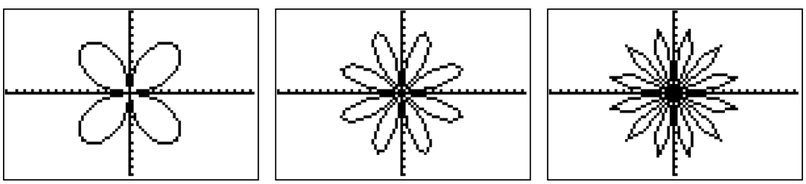

Figure 5.14 Three sample polar roses, all of which have $a = 8$. From left to right, $b = 2$, $b = 4$, and $b = 6$.

5.3 Graphing sequences

Sequences are special types of equations that produce successive terms, rather than a continuous set of values. Each term of a sequence is a number, and it may or may not depend on previous values in the sequence. Here are two simple sequence examples:

- A simple sequence where terms did not depend on the values of previous terms would be the set of positive even numbers, {2, 4, 6, 8, 10, 12,...}. If you want to name terms of the sequence, you can call the first one u_1, the second one u_2, and so on. The nth term is u_n, so the equation for this sequence is $u_n = 2n$: for example, $u_1 = 2$ and $u_3 = 6$.
- A simple sequence where terms depend on the values of previous terms would be a sequence where each term is the sum of the previous two terms. This is a special sequence called a Fibonacci sequence, and it starts {1, 1, 2, 3, 5, 8, 13, 21,...}. In this section, I'll demonstrate graphing the Fibonacci sequence on your calculator.

As in the previous discussions of new graph modes, the "Sequence graphing in a nutshell" sidebar tells you everything you need to know to quickly get started graphing sequences. If you have time to explore two exercises with me, I'll show you how to graph and examine two sequences. The first will be the sequence $u_n = n^2$, where each term is the square of its index. The second sequence is the Fibonacci sequence, a classic and fun example.

Sequence graphing in a nutshell

- To switch to Sequence mode, press MODE, move the cursor to `SEQ`, and press ENTER.
- When you go to the Y= menu, you'll see one setting (`nMin`) and three pairs of items. `u(n)`, `v(n)`, and `w(n)` are the sequences to be graphed, and `u(nMin)`, `v(nMin)`, and `w(nMin)` are lists of initial values, necessary for recursive sequences.
- When you press the XTθn key in `SEQ`, it types an `n` instead of an `X`, because `n` is the independent variable for sequence graphing.
- To type recursive functions, you might need to type `u(n–1)`, `u(n–2)`, `v(n–1)`, and similar expressions. The sequence equation letters `u`, `v`, and `w` can be typed with 2nd 7, 2nd 8, and 2nd 9, respectively.
- As for every other mode, the Zoom menu options discussed in section 3.3.1 can be used to adjust what you see in the graph. `ZoomFit` is particularly useful for sequences.
- The Window menu for Sequence mode has a lot of options. `nMin` and `nMax` control the range of `n` values that are plugged into the sequences. There's no such thing as `nStep`, because `n` always increases by 1. There are also settings for the coordinates of the edges of the screen, as in every graphing mode.
- The `PlotStart` and `PlotStep` options in the Window menu control which values of the sequences are shown on the graph. This won't change the values that are calculated, though, which are controlled by `nMin` and `nMax`.

> **RECURSIVE** A recursive sequence is one where, to find the value of the sequence term u_n, you need to calculate other terms in the sequence as well. *Recursive* means that to calculate those other terms, you need to find still more terms, and the chain continues. The Fibonacci sequence we'll examine in section 5.3.2 is a popular series or sequence for teaching recursion.

Let's start with the easier of our two exercises, the sequence of squares. This will give you a good introduction on how to enter sequences and graph them, as well as how you can use the handy Table tool to see successive values of sequences.

5.3.1 Sequence example: a sequence of squares

Sequences can range from the simple and easily understood to the very complex. We'll start near the easy end, with a sequence containing the squares of the positive integers. Its equation is $u_n = n^2$, so $u_1 = 1$, $u_2 = 4$, $u_3 = 9$, $u_4 = 16$, and so on. It's particularly simple, as far as sequences go, in that you can calculate any term without knowing any other term. To calculate u_9, you plug 9 into $u_n = n^2$ to get $u_9 = 81$.

Start by switching your calculator into Sequence mode, as described in the "Sequence graphing in a nutshell" sidebar. Next, head to the Y= screen, and enter the `u(n)` equation for this sequence:

`u(n)=n²`

Remember, the XTθn key types the independent sequence variable *n* for you. As you can see in figure 5.15, on the left, you should leave `u(nMin)` blank for now, because it's only used for a recursive equation. You might also remember from chapter 3 that the three dots next to `u(n)` mean that you're graphing this sequence as disconnected dots instead of a line.

When you press GRAPH, you'll see the results in the center in figure 5.15. Of course, if you don't have the `ZStandard` window set, things might look different, but choosing `6:ZStandard` under the Zoom menu will swiftly fix that. Because this window is only big enough to show three of the points in this series, you might want to adjust the window. Intuition would tell you that you need to see more of the graph above the current window, so you should increase `Ymax`. The right graph in figure 5.15 exemplifies the window changed to `Ymin` = 0 and `Ymax` = 100.

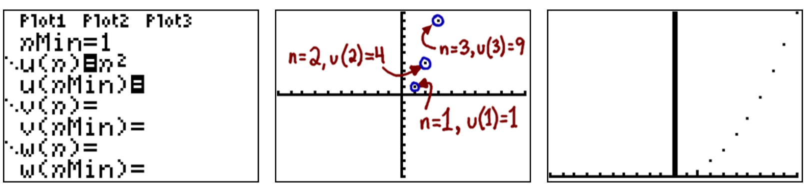

Figure 5.15 Graphing a sequence of squares. On the left, entering the `u(n)` equation. No `u(nMin)` is necessary, because the equation isn't recursive. The center is the result with a `ZStandard` window, and the right is the result with `Ymin` adjusted to 0 and `Ymax` changed to 100.

Using ZoomFit with sequences

The "Sequence graphing in a nutshell" sidebar said that `ZoomFit` is handy for sequences, so let's give that a try. Whether or not you already adjusted the window doesn't matter; go to the Zoom menu and choose `0:ZoomFit`, the tenth item in the list. Your calculator will look at the values of *n*`Min` and *n*`Max`, plus the values of the sequence at the `u(`*n*`)`, `v(`*n*`)`, and/or `w(`*n*`)`, and tailor a good window to the sequence(s) you've graphed.

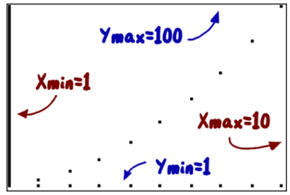

Figure 5.16 Using `ZoomFit` in the Zoom menu to automatically pick a good window to see part of a sequence

Try it for this sequence: `u(`*n*`)=`n^2. With the graph entered, choose `ZoomFit` from the Zoom menu, and you'll immediately see the result, which should look like figure 5.16. If *n*`Max` is set to a large value, this might take a bit longer. The calculator decided to set `Xmin` = *n*`Min` = 1, `Xmax` = *n*`Max` = 10, `Ymin` = $u(1)$ = 1, and `Ymax` = $u(10)$ = 100. This does a good job of showing all the relevant terms in the sequence between *n* = 1 and *n* = 10.

Examining sequence terms: tracing and the Table tool

It's great to visualize sequences on the graphscreen, but often you also want to know exact numerical values of each term. As it often does, your calculator has your needs covered. There are several ways you can get values, but two are particularly fast and easy:

- *Use the Trace feature*—Enter one or more sequences in the Y= menu, graph them, then press [TRACE]. Use [◀] and [▶] to move between sequence terms; the [▼] and [▲] keys switch between different sequences, if you entered more than one. Chapter 3 explains more about using Y=.
- *Use the Table tool*—In section 3.4, you learned how to use the Table tool to see function (rectangular) graph values. The Table tool works for all four graphing modes and is useful for sequences. Graph one or more sequences, and then press [2nd] [GRAPH]. Figure 5.17 shows what the Table tool looks like for the sequence of squares you experimented with in this section. Because the TI-84 Plus C Silver Edition has a larger screen, you may want to switch to the Graph-Table split-screen mode from the Mode menu and then view the table, as illustrated on the right of figure 5.17.

Remember that if you want to change where the table starts or the spacing between values in the table, you can adjust `TblStart` and `ΔTbl` from the TblSetup menu, accessed with [2nd] [WINDOW].

The sequence of squares I showed you in this section was a relatively easy (and nonrecursive) sequence. You'd be missing an important piece of your sequence graphing knowledge if you didn't also go through a recursion exercise.

5.3.2 Sequence example: the Fibonacci series

There are endless examples of recursive functions that are useful in programming, science, and engineering, but the Fibonacci series is a classic and easy-to-understand example. The Fibonacci series was invented by a 13th-century Italian mathematician

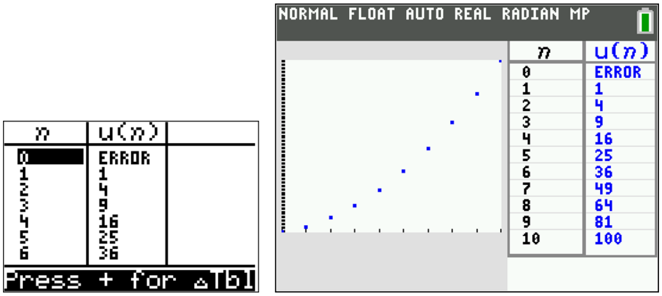

Figure 5.17 Checking sequence values with the Table tool. After graphing a sequence, press 2nd GRAPH to view the table. As with normal graphing, you can modify Table tool settings from the TblSetup (2nd WINDOW) menu. If you use a TI-84 Plus C Silver Edition and switch to the Graph-Table mode from the Mode menu, you'll see the results on the right side instead.

named Leonardo Fibonacci. It's a sequence where the first two terms are 1, and every subsequent term is the sum of the two terms right before it. The base cases (see the "More on recursion" sidebar) of the Fibonacci sequence are $u_1 = 1$ and $u_2 = 1$, and the recursive definition is $u_n = u_{n-1} + u_{n-2}$:

- $u_3 = u_2 + u_1 = 1 + 1 = 2$
- $u_4 = u_3 + u_2 = 2 + 1 = 3$
- $u_5 = u_4 + u_3 = 3 + 2 = 5$
- $u_{10} = u_9 + u_8 = ?$ We can't tell without calculating u_9 and u_8, which requires u_7 and u_6.

In testing the Fibonacci sequence on your calculator, you'll learn how to enter the expression for a recursive series, including teaching your calculator the base case(s). We'll explore how changing the base cases changes every other term in the series. I'll also reiterate the lessons about using `ZoomFit` to see the graphed function better and the Table tool to get exact values.

More on recursion

- Every recursive series has a definition of the *recursive function*. $u_n = u_{n-1} + n$ is a definition that looks back at preceding terms. $u_n = u_{n+1} - n$ is a less-used type of definition that looks forward at following terms.
- Recursion *recursively* calculates values backward (or forward). Because we'd never get values if a recursive function recursed indefinitely, recursive functions need at least one *base case*. When the recursion reaches the base case, it can stop. For example, $u_1 = 1$ would be a useful base case for $u_n = u_{n-1} + n$.

GRAPHING THE FIBONACCI SERIES

Just to enter the Fibonacci series in your calculator's Y= menu, you need to learn two new things. First, you need to learn how to refer to other terms in a series while you're

writing a recursive function definition as `u(n)`. Second, you need to know how to tell your calculator the base cases for the recursive series.

The first problem is defining the function for the recursive series. You already know that each term is `u(n)`, and your calculator plugs in values for *n*. What if you want to refer to the previous term, which in math notation is u_{n-1}? Use `u(n-1)`, typed with [2nd] [7] [(] [XTθn] [−] [1] [)]. As the "Sequence graphing in a nutshell" sidebar explains, [2nd] [7] is the u equation, [XTθn] is the *n* variable in Parametric mode, and the rest is stuff you've seen before. If you want to refer to the term before that, use `u(n-2)`, and so on. For the Fibonacci example, enter `u(n)=u(n-1)+u(n-2)`, as shown on the left in figure 5.18.

> **FORWARD REFERENCES** On your calculator, you can't refer to terms after the current term in a sequence, such as u_{n+1} via `u(n+1)`. This is invalid syntax. If you try, you'll get an `ERR:DOMAIN` error. You can only refer to terms before the current term.

Next, you need to enter the base cases:

- If you have one base case, you can enter it as a single number next to `u(nMin)` (and `v(nMin)` and `w(nMin)`, if you have two or three sequences). For example, you could enter `u(nMin)=5`.
- If you have more than one base case, which happens when you have a recursive function that refers to the u_{n-2} term or further back, you need to enter the base cases as a list. For two base cases, you might enter `{4,2}`; for three, `{4,2,1}`.

For our Fibonacci exercise, we have two base cases: enter the list `{1,1}` for `u(nMin)`, because those are the first two terms in the Fibonacci sequence. The curly braces are [2nd] [(] and [2nd] [)], as chapter 4 explained.

You have entered `u(n)` and `u(nMin)`, so you're ready to graph the Fibonacci series by pressing [GRAPH]. In all likelihood, the graph won't look quite right. An easy fix is the `ZoomFit` tool we looked at in the first sequence example. Press [ZOOM], and choose `0:ZoomFit`. Much better! Now you should see the plot shown on the right in figure 5.18.

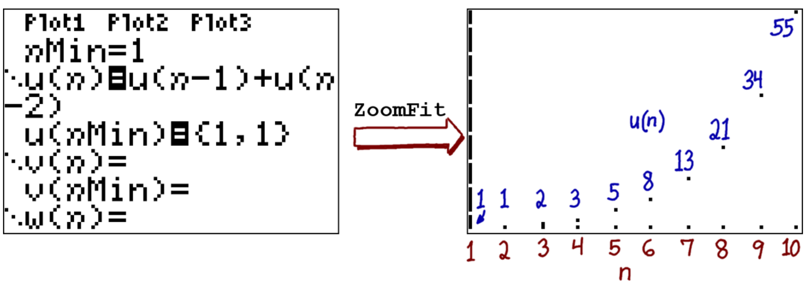

Figure 5.18 Graphing the first 10 terms of the Fibonacci series on your calculator in Sequence mode. The left shows entering the recursive function and the base cases. The right is the graph after you adjust the window with `ZoomFit`.

VIEWING FIBONACCI SERIES VALUES

You could trace along that graph, but an easier way to examine the values of the series is to look at the table. Switch into Table mode with [2nd] [GRAPH], and you'll see values of the Fibonacci sequence side-by-side with their indices (n), as figure 5.19 illustrates. You can use the arrow keys to scroll up and down, and you'll soon discover that you can't go to indices before $n = 1$, because there are no terms in the Fibonacci series before $u_1 = 1$. You can have negative indices in sequences on your calculator; the lowest possible n value is whatever `nMin` is set to. If you want to scroll way down (or way up) the table, you should press [2nd] [WINDOW] and adjust the `TblStart` variable.

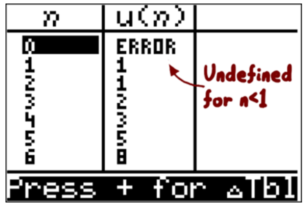

Figure 5.19 Viewing the table of indices and values for the recursive Fibonacci sequence

You've learned just about everything you need to know to use Sequence graphing mode effectively on your calculator to explore recursive and nonrecursive series. In fact, between this chapter and chapter 3, you know all four graphing modes that your calculator offers: Function, Parametric, Polar, and Sequence modes. But did you know you can draw on top of graphs and save those annotated graphs for later? Drawing and saving graphs and graph settings are the subject of the remainder of this chapter.

5.4 Drawing on graphs

Graphing calculators are wonderful math tools. In a few seconds you can go from staring at an equation in a textbook to exploring a graph of the function, without having to tediously draw out the graph by hand. Say you find an interesting point in such a graph. You want to circle the point, add some text next to it to explain what it is, and save the graph to show to your teacher later. At this point, you'll probably end up trying to sketch the graph on a page in a notebook and annotate it by hand. After you read this section, though, you'll know how to draw those annotations directly on the graph. By the end of section 5.5, you'll also have learned how to save the annotated graph in your calculator's memory.

Your calculator includes a host of drawing tools, including 10 ways to draw lines, circles, text, and even pieces of functions, and 6 ways to draw points. You can draw directly on the graphscreen, or you can run drawing functions as homescreen functions. In this section, I'll show you how to draw on top of graphs. You'll learn to use these functions directly on the graph and which ones can also be used on the homescreen. I'll also show you the three graphing tools that are related to graphed functions: `DrawInv`, `DrawF`, and `Shade(`. By the time you get to the end of this section, you'll be well versed in drawing on your calculator. And because I'm nothing if not a realist, I know that many of you will end up using these tools to sketch drawings of your own. I did it too back in the day.

Let's start with drawing functions like `Horizontal`, `Line(`, and `Text(` used on the graphscreen and homescreen.

5.4.1 Graphscreen drawing tools

No matter what you're trying to draw on your calculator, from graph annotation to geometry diagrams to doodles, your calculator has a full complement of tools to help you. The best way to learn to use the drawing tools is to play around with them, so I'm going to cover the high level of using the tools in general and let you experiment for yourself. To guide you, I've created tables 5.1 and 5.2 (later in this section) explaining each of the tools.

With few exceptions, most drawing tools can be used one of two ways:

- Go to the graphscreen by pressing [GRAPH]. Optionally, you can graph equation(s) first. From the graphscreen, press [2nd] [PRGM] to get to the Draw menu, pick a tool, and use it. Continue until you're happy with your creation.
- Go to the homescreen and enter drawing commands as functions, like `round(` and `gcd(` and all their friends. This requires memorizing the *arguments* to each function, the values you put in parentheses after the function names. Chapter 2 explained how to use functions on the homescreen.

I'll start you with drawing by example, namely graphing a parabola and annotating that graph. We'll then touch briefly on homescreen drawing commands and conclude this section with a table of the drawing tools you'll use most frequently. If you have a TI-84 Plus C Silver Edition, you may also want to refer to section 12.3, which discusses the differences between drawing on the older and newer calculators.

USING DRAWING TOOLS ON THE GRAPHSCREEN: ANNOTATING A PARABOLA

The easiest way to draw on your calculator is to use its drawing tools directly on the graphscreen. You can draw lines, circles, text, points, and more. To select any drawing tool, start on the graphscreen, press [2nd] [PRGM] to access the Draw menu, and pick the tool you want. The Draw menu has three tabs—DRAW, POINTS, and STO—as you can see from figure 5.20. You'll learn about the STO tab in section 5.5. On the TI-84 Plus C Silver Edition, there's a fourth tab called Background, but you won't need to use that unless you're writing a program.

When you pick a drawing tool from the Draw menu, that tool will remain active until you pick another one. You can draw multiple lines, circles, strings of text, and more without needing to keep reselecting the same tool from the Draw menu. You'll use that technique in this section's example: annotating a parabola.

Start with a parabola by switching to Function mode (by accessing the Mode menu), entering $Y_1=5-(X-2)^2$ in the Y= menu, and pressing [GRAPH]. Doesn't look like figure 5.21? No problem; go to the Zoom menu, and choose `6:ZStandard`. You can

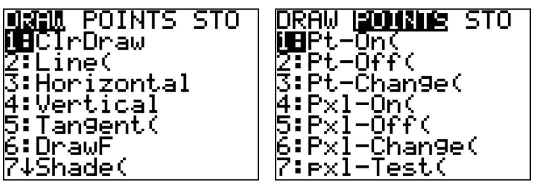

Figure 5.20 The DRAW and POINTS tabs of the Draw menu, containing general tools (left) and point-drawing tools (right). The STO tab contains ways to save and restore graph settings and pictures, as you'll see in section 5.5.

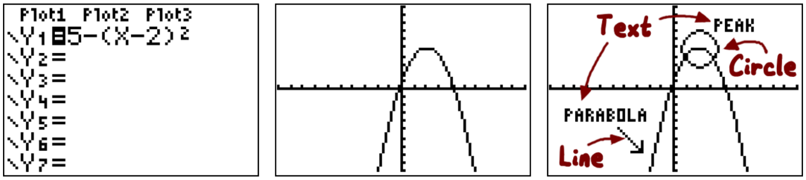

Figure 5.21 Graphing a parabola and annotating it. `PEAK` and `PARABOLA` are drawn with the `Text` tool, the circle is a `Circle`, and the arrow is three `Line`s. The left side shows the equation, and the middle shows the graph before being annotated.

always turn the axes on and off and modify other graph format options using the [2nd] [ZOOM] keys.

Now let's try some annotations:

1. Try circling the peak. From the graph, press [2nd] [PRGM] to open the Draw menu, and scroll down to `9:Circle(`. Move the cursor over the peak, press [ENTER] to set the circle's center, and then move the cursor to a point where you want the edge of the circle to be and press [ENTER] again. A circle!
2. Write out `PEAK`, as in figure 5.21. Go to `0:Text(` in the Draw menu, and then use the arrow keys to move the cursor. Place it at the top-left corner of where you want the word `PEAK` to appear, and type `PEAK`. Remember that to type a string of letters, you press [2nd] [ALPHA] to engage Alpha-Lock, and then press the keys that have the letters you want printed over them.
3. You can type `PARABOLA` without returning to the Draw menu. Move the cursor to a new position with the arrow keys, and type `PARABOLA`.
4. Create the arrow pointing to the parabola in figure 5.21. Go to the Draw menu, and select the `2:Line(` tool. For each line, move the cursor to the first endpoint, press [ENTER], move the cursor to the second endpoint, and press [ENTER] again. The tool you select stays active until you select another tool, so you don't need to return to the Draw menu before each new line. The arrow in figure 5.21 is three lines.

While you were drawing, you might have been frustrated that the coordinates at the bottom of the screen were getting in your way. If you don't mind them while you're drawing, but you want them to go away when you finish drawing, press [CLEAR]. Don't worry; it won't clear the screen. At worst, you might accidentally quit to the homescreen, at which point you need only press [GRAPH] to return to the drawing. If you don't want coordinates to show at all while you draw, select `CoordOff` from the Graph Format menu, [2nd] [ZOOM]. If you're on a TI-84 Plus C Silver Edition, you could press [GRAPH] in the middle of drawing any shape to access the Style menu, as indicated by the onscreen `Style` button above the [GRAPH] key.

CLEARING THE GRAPHSCREEN: CLRDRAW

The `1:ClrDraw` command from the Draw tab of the Draw menu clears the entire graphscreen. If you have the axes or grid enabled, it redraws them as well and then rerenders any functions you've graphed. Press [GRAPH], and then go to the Draw menu

with 2nd PRGM, and choose `1:ClrDraw`. If you're on the homescreen, you can enter `ClrDraw` as a command on a line by itself and press ENTER.

Be aware that zooming a graph, panning left or right, changing the window, or changing or removing an existing graphed function will also erase the graphscreen. Adding a new function will *not* erase annotations you've drawn, as long as you don't change existing functions.

> **SKETCHING ON A BLANK SCREEN** Want a blank screen for your doodles, diagrams, and sketches? Turn off the axes and grid by making sure `AxesOff` and `GridOff` are selected in the Graph Format menu, under 2nd ZOOM. Don't forget how to turn them on again if you need to graph, but remember that graphing and modifying Graph Format options will erase anything you've already drawn!

5.4.2 Using drawing tools on the homescreen

It's easy to use your calculator's drawing tools on the graphscreen, but sometimes you need more precision than you can get by using the tools directly on the graphscreen. Or perhaps you need to draw lines or circles that are partially offscreen. In this case, you'll want to use the drawing tools from the homescreen.

Let's go through an exercise, because that's the easiest way to learn anything. You'll draw a circle, a horizontal line, and a vertical line from the homescreen. Here are the steps:

1. Remove any equations from the Y= menu. You can also turn off the axes with `AxesOff` from the Format menu (2nd ZOOM) if you want. You may want to use `ZStandard` and then `ZSquare` from the Zoom menu to make your results match my screenshots.
2. Use `ClrDraw` from the Draw menu to start with a blank graphscreen.
3. From the homescreen, issue the commands `Circle(2,4,10)` to draw a circle centered at (2,4) of radius 10; you can see the command in figure 5.22 or figure 5.23. If you're using a TI-84 Plus C Silver Edition, use the command `Circle(2,4,10,RED)`; you can find `RED` under VARS ▶ ▶.
4. Use the `Horizontal -6` and `Vertical -8` options to draw two lines tangent to the circle. You should see something that matches the right side in figure 5.22 or figure 5.23. If you have a TI-84 Plus C Silver Edition, try `Vertical -8,GREEN` instead of `Vertical -8`.

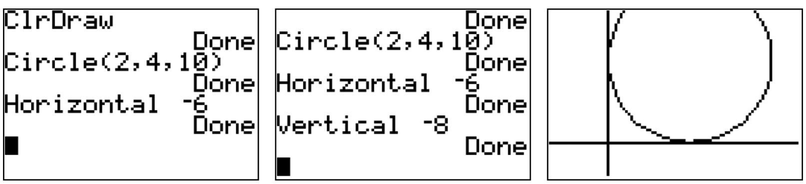

Figure 5.22 Using drawing commands from the homescreen to draw a circle, a horizontal line, and a vertical line. For this example, I turned off the axes with `AxesOff` and used `ZStandard` and `ZSquare` to set up the window.

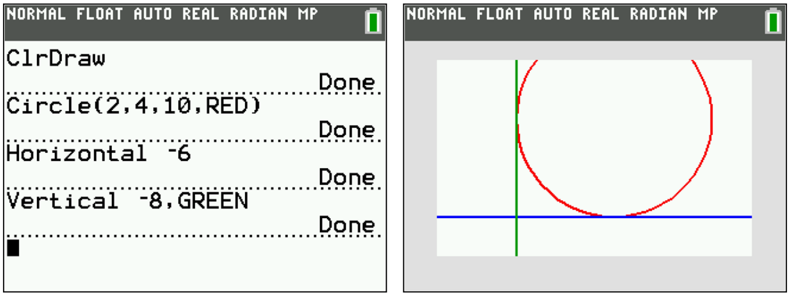

Figure 5.23 The same set of drawing commands as figure 5.22 on the TI-84 Plus C Silver Edition. I used colors in two of the commands, found in the Color tab of the Vars menu.

For the full list of drawing commands, look at tables 5.1 and 5.2. Table 5.1 contains most of the interesting commands from the Draw menu, and table 5.2 lists the commands from the Points tab.

Table 5.1 Drawing functions in the Draw tab of the Draw menu. You must first go to the graphscreen, press [2nd] [PRGM] and select the function you want, and then use it. You can also go to the homescreen instead to use many of these functions. If you have a TI-84 Plus C Silver Edition, you should instead refer to table 12.2 in section 12.3.

Command	What it does	Homescreen syntax
`Line(`	Draws a line between two points. On the graphscreen, move the cursor to the first point, press [ENTER]; move it to the second point, and press [ENTER] again.	`Line(`<x_1>,<y_1>,<x_2>,<y_2>`)` Example: `Line(`$^{-}$`1,0,8.2,8)`
`Horizontal`	Draws a horizontal line across the whole screen. Move the line to where you want it, and press [ENTER] to make it permanent.	`Horizontal` <y coordinate> Example: `Horizontal 1.5`
`Vertical`	Draws a vertical line down the whole screen. Move the line to where you want it, and press [ENTER] to make it permanent.	`Vertical` <x coordinate> Example: `Vertical` $^{-}$`4`
`Tangent(`	Draws a line tangent to a given function at a given *x* point. Always assumes that functions are rectangular (FUNC) functions, even if the calculator isn't in Function mode. On the graphscreen, you select the function and then the point.	`Tangent(`<func>,<x coord>`)` Example: `Tangent(3X`^{2}`,1.25)`
`Circle(`	Draws a circle with a center and radius. On the graphscreen, select the center, press [ENTER], and then choose a point on the edge of the circle.	`Circle(`<center >, <center y>,<radius>`)` Example: `Circle(2,`$^{-}$`1,4)`

Table 5.1 Drawing functions in the Draw tab of the Draw menu. You must first go to the graphscreen, press 2nd PRGM and select the function you want, and then use it. You can also go to the homescreen instead to use many of these functions. If you have a TI-84 Plus C Silver Edition, you should instead refer to table 12.2 in section 12.3. *(continued)*

Command	What it does	Homescreen syntax
`Text(`	Writes out text. Move the cursor where you want the top-left corner of the text to be, and start typing (no need to press ENTER). Remember to press ALPHA if you want to type letters. For entering `Text(` on the homescreen, row and column are pixels from the top-left of the screen, which is row 0, column 0. The bottom-right is row 62, column 94.	`Text(<row>,<column>, "STRING")` Example: `Text(57,1,"GRAPH TITLE")`
`Pen`	Somewhat like drawing with an Etch-a-Sketch. Move the cursor to where you want to start, press ENTER to put the "pen" down, and move the "pen" to draw a black line behind it. You can press ENTER to lift the "pen" and move it to a new spot to draw again.	(You can't use the `Pen` as a homescreen command.)

Table 5.2 Point and pixel commands, all in the Points tab of the Draw menu

Command	What it does	Homescreen syntax

`Pt-On(`
`Pt-Off(`
`Pt-Change(`
	Once you enable this tool, you can freely move the cursor around the graphscreen. Every time you press ENTER, it will turn the point on (to black), turn it off (to white), or change it. From the homescreen, this command uses (*x*,*y*) coordinates. If you omit `<type>`, it draws a single dot. You can enter 2 for type for a square and 3 for a cross.	`Pt-On(<x>,<y >,[<type])`
		`Pt-Off(<x>,<y >,[<type])`
		`Pt-Change(<x>,<y>,[<type])`
		Example: `Pt-On(5,1.2)`

`Pxl-On(`
`Pxl-Off(`
`Pxl-Change(`
	Can't be drawn on the graphscreen; only for use as a homescreen function. Like `Text(`, takes pixel coordinates. Row and column are pixels from the top-left of the screen, which is row 0, column 0. The bottom-right is row 62, column 94.	`Pxl-On(<row>,<column>)`
		`Pxl-Off(<row>,<column>)`
		`Pxl-Change(<row>,<column>)`
		Example: `Pxl-Off(30,52)`

There are three more drawing commands that we haven't discussed yet in the Draw tab of the Draw menu, all of which work with graphed equations. You'll learn how to use those before we move on to the final topic of the chapter, saving and restoring pictures and graph settings.

5.4.3 Drawing graphlike functions: DrawInv, DrawF, and Shade

Three of the tools in the Draw menu (2nd PRGM) take functions as arguments and draw something based on equations. All three work only in Function mode or act as if the calculator is in Function mode regardless of the current settings. These three functions are as follows:

- `DrawInv`—Draws the inverse of a `Y=` rectangular equation. Useful for drawing `X=` equations (see the "Graphing X= equations" sidebar for more information). This function takes one argument: the equation to graph the inverse of. For example: `DrawInv X²+3`.
- `DrawF`—Graphs a function, just as if you had entered it in the Y= menu. This function works only with rectangular/function (`Y=`) equations, even if the graphing mode is set to one of the other modes. Like `DrawInv`, it takes a single argument: an equation to graph. For example, `DrawF √(X)` would be equivalent to setting `Y₁=√(X)`.
- `Shade`—Draws a solid or hashed shade between two functions (or y coordinates). It takes at least two arguments: the two functions (or y coordinates) bounding the shaded area. For example, `Shade(X²,10)` shades the area between $y = x^2$ and $y = 10$ in solid black. You can also choose minimum and maximum x coordinates on the shaded area, so `Shade(X²,10,0,2)` only shades between $x = 0$ and $x = 2$.

> **Graphing X= equations**
>
> Many graphing-calculator users struggle to find a way to graph equations of the form $x = f(y)$, such as $x = y^2$ or $x = 1 + 5\sin(y)$. If you're looking for a way to graph `X=` equations, there are *two* possible methods:
>
> - `DrawInv`—If your calculator is already in Function (Rectangular) graphing mode, use the `DrawInv` command from the Draw menu. `DrawInv` takes one argument: the function to graph, with `Y`s replaced by `X`s. In other words, `DrawInv X²` draws the equation $x = y^2$.
> - *Parametric mode*—Take your $x = f(y)$ equation, such as `X=1+5sin(Y)`, and replace the `Y`s with `T`s. Enter your equation as x_{1T}, such as `X1T=1+5sin(T)`, and set `Y1T=T`. You'll also need to adjust `Tmin` to `Ymin` and `Tmax` to `Ymax`.

Except for `DrawInv`'s use as a way to graph `X=` equations, you probably won't find yourself using these three tools that often. Nevertheless, for the sake of completeness, I'd like to show you a brief exercise that uses all three of these drawing tools. You'll use `DrawF` and `DrawInv` to graph `Y=3sin(X)` and `X=3sin(Y)` and then shade between `Y=3sin(X)` and `Y=3sin(X)-3` using the `Shade(` command.

To set up, press MODE and change your calculator to Function mode, if it's not already in it. Turn on the axes from the Graph Format menu (2nd ZOOM) if they're not already on, and select `6:ZStandard` from the Zoom menu. Finally, if you still have anything on the graphscreen, clear any entered equations from the Y= menu,

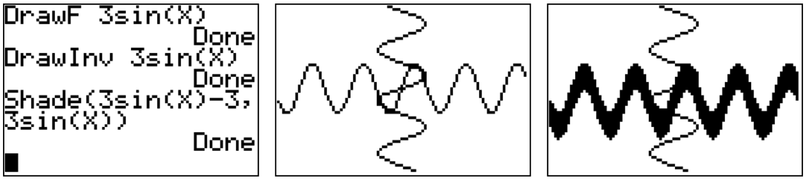

Figure 5.24 Using the `DrawF`, `DrawInv`, and `Shade(` commands. The left shows the three commands to enter, the center shows the effect of just `DrawF` and `DrawInv`, and the right is the result after the `Shade(` command.

and, if necessary, select `1:ClrDraw` from the Draw menu ([2nd] [PRGM]). Next, follow these steps:

1. Select `6:DrawF` from the Draw menu, and add `3sin(X)` as an argument. You'll end up with `DrawF 3sin(X)`, as in figure 5.24. Press [ENTER] to execute the command.
2. Choose `8:DrawInv` from the Draw menu, and add `3sin(X)` again, to get `DrawInv 3sin(X)`. This graphs $x = 3\sin(y)$; from the `DrawF` and `DrawInv` commands, you'll get the screenshot in the center of figure 5.24.
3. Use the `Shade(` command from the Draw menu to shade between $y = 3\sin(x) - 3$ and $y = 3\sin(x)$ with `Shade(3sin(X)-3,3sin(X))`. Press [ENTER], and when the calculator finishes drawing, you'll see something that resembles the right side of figure 5.24.

As with all the other drawing tools, and in fact nearly every skill you've learned so far, I encourage you to experiment with these three new drawing features to see how they can help you with your academic work and your own drawings. But what if you want to save the results of your drawing efforts? In the next section, you'll learn to store snapshots of the graphscreen and restore them, as well as save and restore graph settings for later use.

5.5 Saving graph settings and pictures

The final skills of this chapter are among the easiest of the new graphing and drawing skills, and thus I saved them until the end to relax your brain after the rigors of graphing and drawing. In addition, there's no point saving pictures of the graphscreen and your graph settings if you have neither graphs nor pictures. In a nutshell, this section will show you how to take a snapshot (or picture or screenshot, if you prefer) of the graphscreen and save it into your calculator's memory. You can later restore one of those pictures to return the graphscreen to how it looked when you took the picture, regardless of any new graphs, drawings, mode changes, and format changes that you made in the meantime. This section will also explain graph databases (GDBs), a way to store all currently graphed equations plus your Graph Format settings and graph window into memory. You can then later restore a GDB to return your graphs and settings to the way they were when you stored the GDB.

Saving and recalling pictures (pics) is useful whenever you've made an annotated graph or a drawing that you want to store for later. Pictures are the first topic we'll

examine. The second is saving and recalling GDBs, which is handy when you have your graphed functions and graph format settings exactly as you want them and want to be able to later switch back to those settings.

Let's get right to it with a look at your calculator's picture variables and features.

5.5.1 Saving and recalling picture variables

Just as your calculator has numerical variables for storing numbers, list variables for storing lists, and matrix variables for storing matrices, it has a set of 10 picture variables for storing pictures or snapshots of the graphscreen. Named `Pic1` through `Pic9` plus `Pic0`, each is a location in your calculator's memory, and each can store a full image of the graphscreen.

> **WHY NOT PIC0 THROUGH PIC9?** In section 5.5.1, I refer to "`Pic1` through `Pic9` plus `Pic0`" instead of just saying "`Pic0` through `Pic9`." This isn't an error or an attempt to confuse you. Because your calculator thinks of `Pic0` as `Pic10`, it's the last of the 10 picture variables, after `Pic9`.

The `StorePic` and `RecallPic` commands, which respectively store and recall picture variables, can be found in the STO tab of the Draw menu. To access this menu, shown in figure 5.25, press [2nd] [PRGM] [▶] [▶]. Once you choose `StorePic` or `RecallPic`, the command is pasted to the homescreen. Add a `1`, `2`, `3`, …, `9`, or `0` after either `StorePic` or `RecallPic` to save or open that picture number.

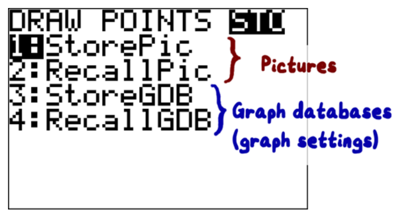

Figure 5.25 The STO tab of the Draw menu, from which you can save and recall both pictures and graph settings (GDBs)

> **PICTURE CAVEATS** If you `StorePic` a picture that already exists, your calculator will overwrite the old picture without warning you. If you try to `RecallPic` a picture that doesn't exist, your calculator will show an `ERR:UNDEFINED` message.

If you want to see a list of which picture variables you have on your calculator, you can press [2nd] [+] to access the Memory menu, then choose `2:Mem Mgmt/Del…`, and choose `8:Pic…`. You'll see a list of any picture variables on your calculator, and you can archive, unarchive, and delete pictures from here. Press [DEL] next to any picture to delete it, which saves memory (RAM). If you archive a picture by pressing [ENTER] next to it, which adds an asterisk (`*`), then it won't take up RAM and will survive a RAM clear. But you can't `StorePic` or `RecallPic` that variable until you unarchive it by returning to the Memory menu and pressing [ENTER] next to it again.

To demonstrate saving and recalling pictures, I drew a sketch on the graphscreen, as shown on the left in figure 5.26. Next, I cleared the graphscreen with `ClrDraw` and then recalled the picture that I saved, as shown in the center in figure 5.26. When I pressed [GRAPH], the sketch I had drawn was restored.

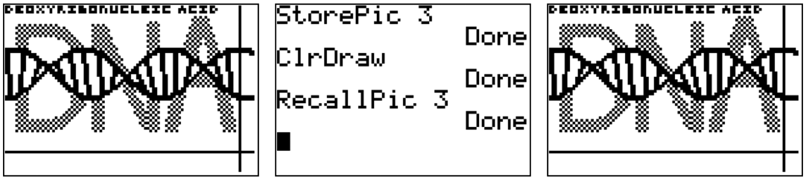

Figure 5.26 Demonstrating how `StorePic` and `RecallPic` let you save a snapshot of graphs, annotations, or drawings on the graphscreen. Here, I store the DNA doodle I drew, clear the screen (erasing the doodle), and then use `RecallPic` to restore the sketch.

What if you want to save not the exact contents of the graphscreen but the equations that you have currently graphed, plus the window and format settings? Then you should use a GDB.

5.5.2 *Saving and recalling graph databases*

A graph database, or GDB, is a somewhat different animal than a picture variable. There are 10 of them, `GDB1` through `GDB9` plus `GDB0`, and they're located in your calculator's memory. But instead of holding an exact snapshot of the pixels on the graphscreen, GDBs hold all the following:

- Any equations you have entered in the Y= menu, for all four modes.
- The current graph mode.
- The Graph Format settings from the Format menu, including whether the axes, the grid, and coordinates are on or off.
- The window variables, including `Xmin`, `Xmax`, `Ymin`, `Ymax`, and any other values in WINDOW.
- On the TI-84 Plus C Silver Edition, it also saves the colors of each graphed equation, the axes color, the grid color, and the color of the border around the graph. It does *not* save the background picture or color.

Saving and recalling GDBs use the `StoreGDB` and `RecallGDB` functions from the STO tab of the Draw menu, as shown in figure 5.25. You follow `StoreGDB` and `RecallGDB` with a number from 0 to 9, exactly as with `StorePic` and `RecallPic`. I showed you how to list the picture variables on your calculator and archive and delete them from the Memory menu. You can list, delete, and archive the GDB variables nearly the exact same way: press 2nd + to access the Memory menu, then choose `2:Mem Mgmt/Del...`, and choose `9:GDB...`.

Figure 5.27 shows an example of storing a GDB, messing with Graph Format settings and graphed equations, and then recalling the original GDB. By recalling `GDB4`, the initial set of functions and settings from the left in figure 5.22 is restored, undoing the changes I made to the graph format and the window settings.

And now we've reached the end of what you need to know for advanced usage of your calculator's drawing and graphing features! From here on, we'll focus on advanced

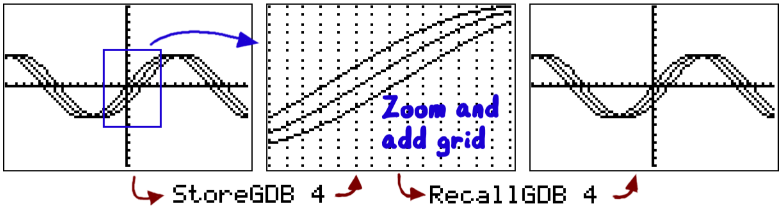

Figure 5.27 Starting with the graph on the left, the `StoreGDB 4` command stores the graphed equations and graph settings to graph database `GDB4`. After I turn the axes off and the grid on and change the window settings, the graph changes to what you see in the center. The `RecallGDB 4` command restores it to the right view, just as it was when the `StoreGDB 4` command was issued.

math features, although some of the discussion, such as statistics, will include use of plots and graphs on the graphscreen.

5.6 Summary

In this chapter, you learned advanced skills for graphing and drawing. We started with the three graphing modes that chapter 3 didn't explore: Parametric mode, Polar mode, and Sequence mode. You learned how Parametric mode lets you express and graph functions that would be impossible in Function or Rectangular mode. You explored examples of polar functions, from circles and spirals to rose-shaped graphs. We worked through sequence graphing exercises of a recursive series like the Fibonacci series and a nonrecursive series like the squares of the positive integers.

In the latter half of the chapter, I showed you the many drawing and annotation tools that your calculator offers. From ways to mark up graphed equations to shade functions and graph `X=` equations, there's not much you can't draw on your calculator's graphscreen. You even learned how you can use the graphscreen as a creative canvas for freehand sketches and diagrams. We concluded with a look at the tools for storing and recalling pictures, which store the pixel-by-pixel contents of the graphscreen; and GDBs, which hold the currently graphed equations and graph settings.

As we move forward into the rest of part 2 of this book, you'll be graduating to the realm of precalculus and calculus. Chapter 6 focuses on precalculus features of your graphing calculator that you haven't explored yet. You'll look at complex numbers, the nitty-gritty of trigonometry, limits, and logarithms. Onward!

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